Simple Concepts and Estimates…

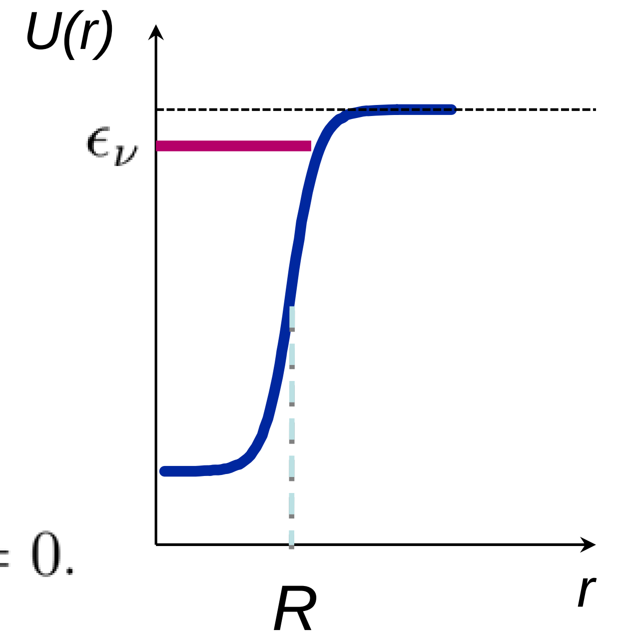
Radii of halo systems

Riisager et al., Nucl. Phys. . A548, 393 (1992)
Misu et al., Nuclear Physics A614, 44 (1997)

The usual starting point: one body Schrödinger equation:

$$\left[\nabla^2 - \frac{2m}{\hbar^2}U(r) - \kappa_\nu^2\right]\psi_\nu(r) = 0$$

$$\kappa_\nu = \sqrt{-2m\epsilon_\nu/\hbar^2}$$



at large distances…

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \kappa_\nu^2 - \frac{\ell(\ell+1)}{r^2}\right]R_{\ell\nu}(r) = 0.$$

asymptotically… $R_{\ell\nu}(r) = B_\ell h_\ell^+(i\kappa_\nu r)$

We are interested in the expectation value:

$$\langle \ell\Lambda\nu|r^n|\ell'\Lambda\nu\rangle \equiv \int_0^\infty r^{n+2}R^*_{\ell\Lambda\nu}(r)R_{\ell'\Lambda\nu}(r)dr = I_{n\ell\ell'\Lambda\nu} + O_{n\ell\ell'\nu}$$

inner contribution (r<R)

outer contribution (r>R)

The inner integral is always finite. The outer integral can be written as:

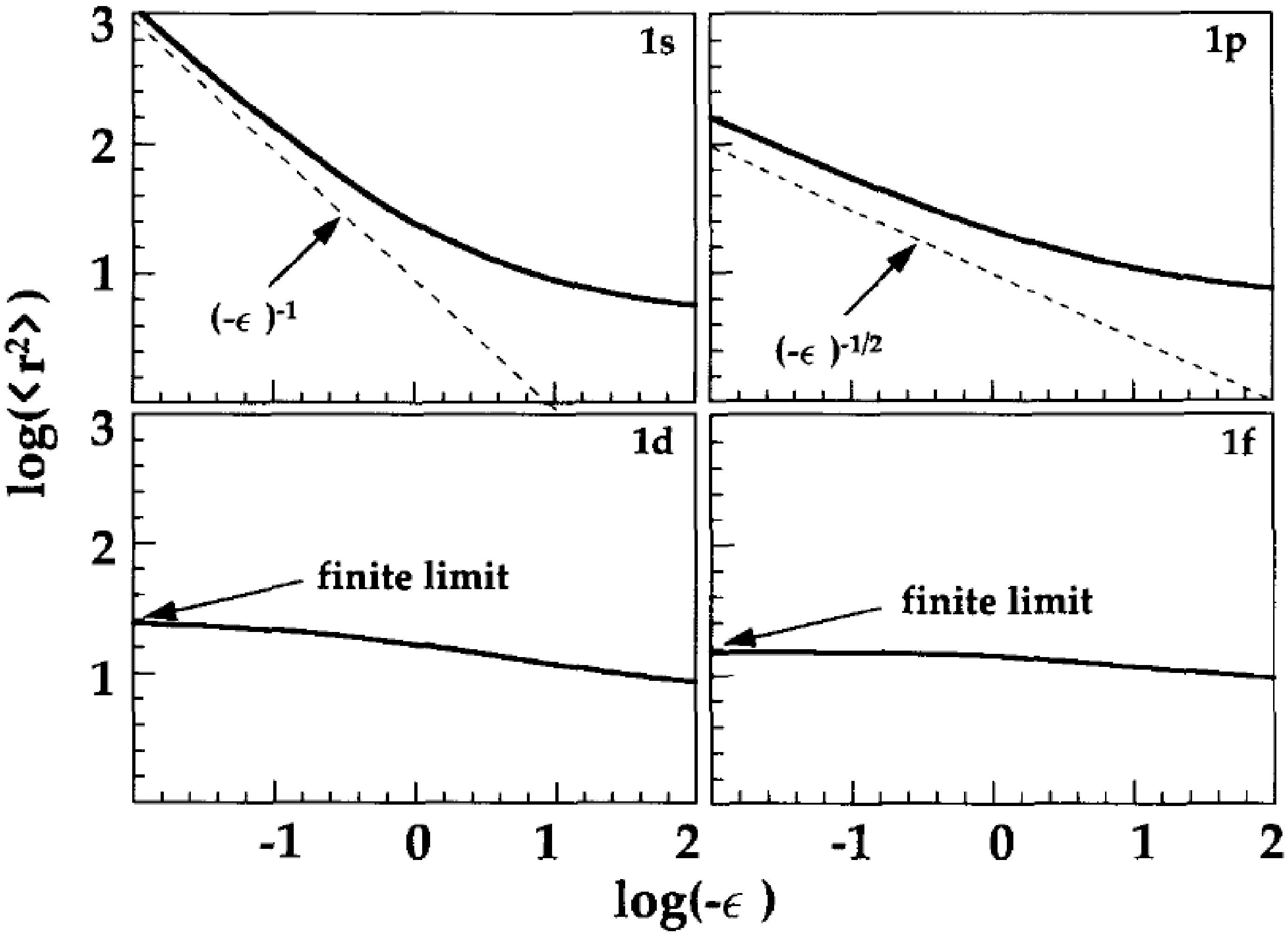
$$\begin{aligned} O_{n\ell\ell'\nu} &= \int_R^\infty r^{n+2} B_\ell^* B_{\ell'} h_\ell^{+*}(i\kappa_\nu r) h_{\ell'}^{+}(i\kappa_\nu r) dr \\ &= B_\ell^* B_{\ell'} \kappa_\nu^{-(n+3)} \int_{R\kappa_\nu}^\infty h_\ell^{+*}(ix) h_{\ell'}^{+}(ix) x^{n+2} dx \end{aligned}$$

In the limit of a very weak binding, one can use the asymptotic expressions for the Hankel functions. This yields:

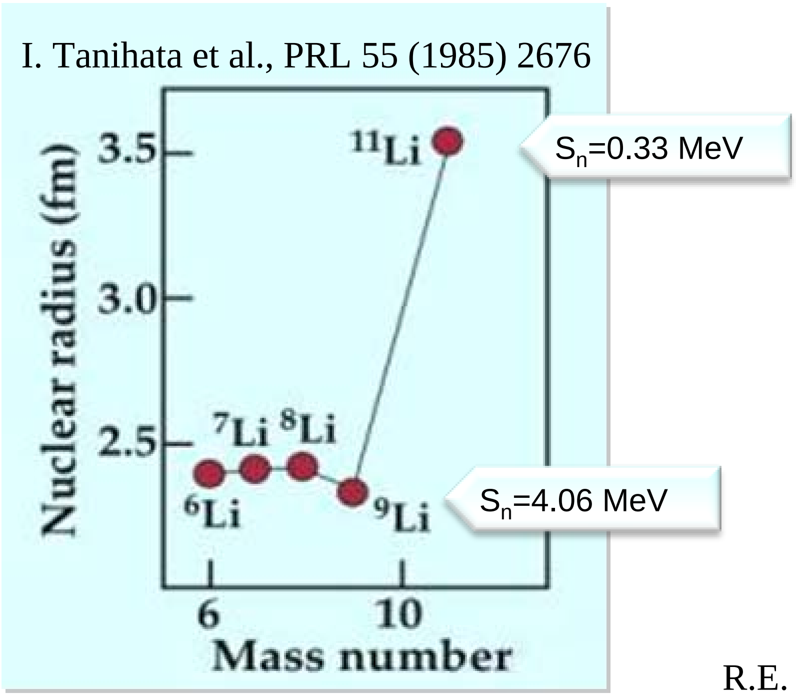
$$B_\ell \approx \frac{i^{\ell+1}}{1 \times 3 \times ...(2\ell-1)} R_{\ell\nu}(R)(R\kappa_\nu)^{\ell+1}$$

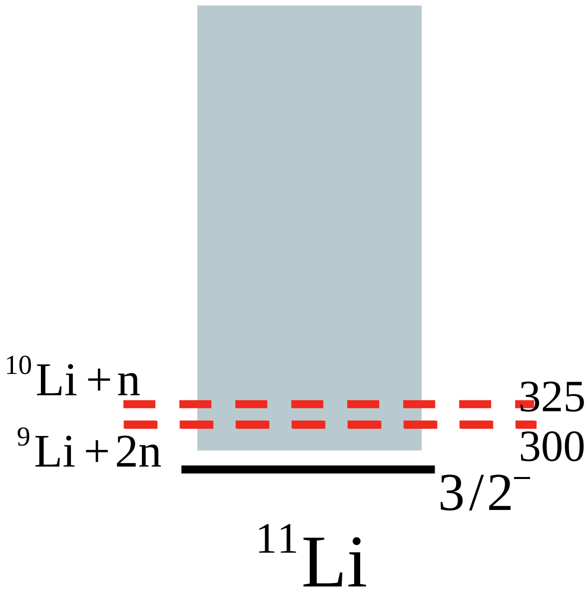
$$\begin{array}{lll} n > \ell + \ell' - 1: & O_{n\ell\ell'\nu} & \text{diverges as } (-\epsilon_\nu)^{(\ell+\ell'-n-1)/2}, \\ n = \ell + \ell' - 1: & O_{n\ell\ell'\nu} & \text{diverges as } -\dfrac{1}{2}\ln(-\epsilon_\nu), \\ n < \ell + \ell' - 1: & O_{n\ell\ell'\nu} & \text{remains finite} \end{array}$$

$$\ell = 0: \quad \langle r^2 \rangle \quad \text{diverges as } (-\epsilon_\nu)^{-1},$$
$$\ell = 1: \quad \langle r^2 \rangle \quad \text{diverges as } (-\epsilon_\nu)^{-1/2}$$

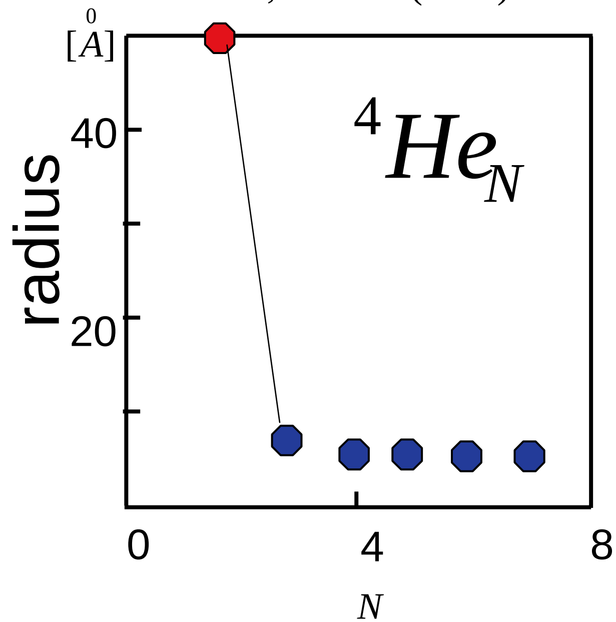


If pairing is present, this picture changes:
K. Bennaceur et al., Phys. Lett. B496, 154 (2000)



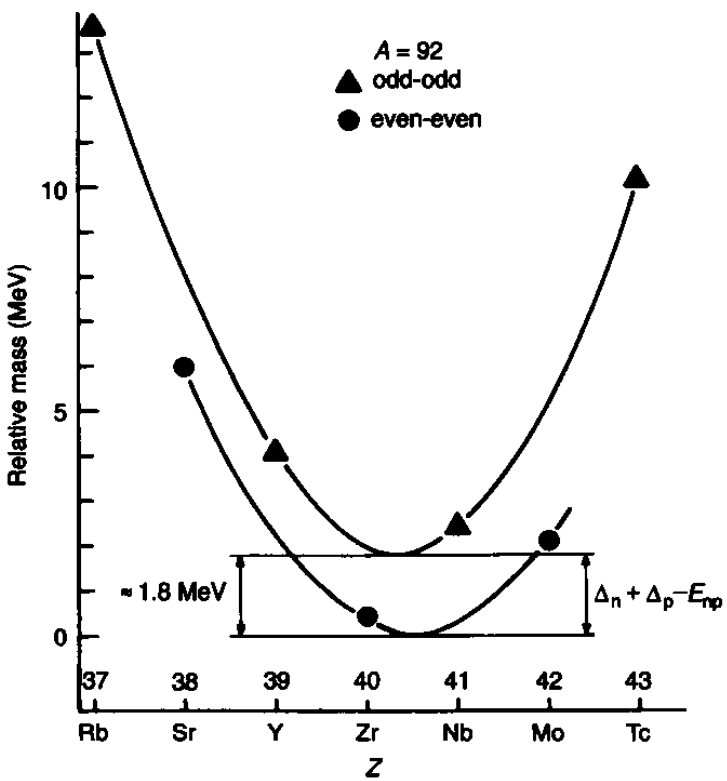


R.E. Grisenti et al., PRL 85 (2000) 2284



Odd-even mass difference

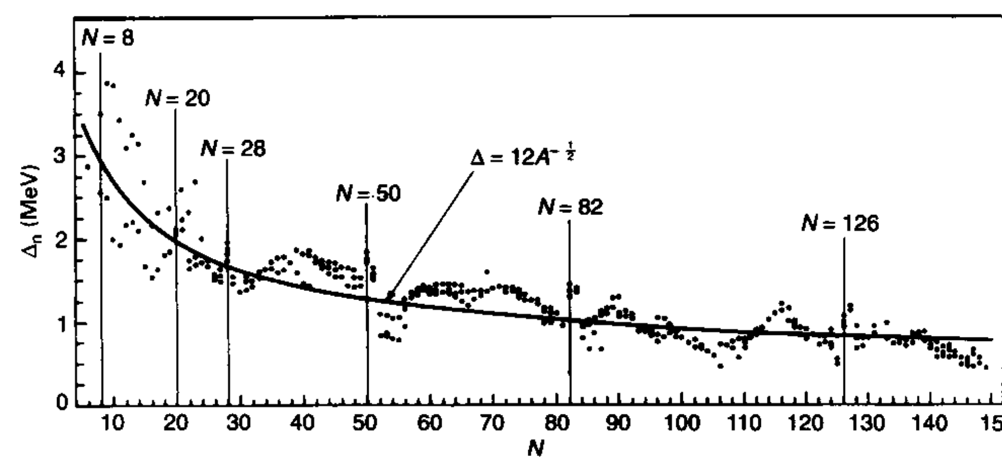
The semi-empirical mass formula and nuclear stability



$$\Delta_n = B(N,Z) - \frac{B(N+1,Z) + B(N-1,Z)}{2}$$

$$\Delta_p = B(N,Z) - \frac{B(N,Z+1) + B(N,Z-1)}{2}$$

A common phenomenon in mesoscopic systems!



Pairing and separation energy

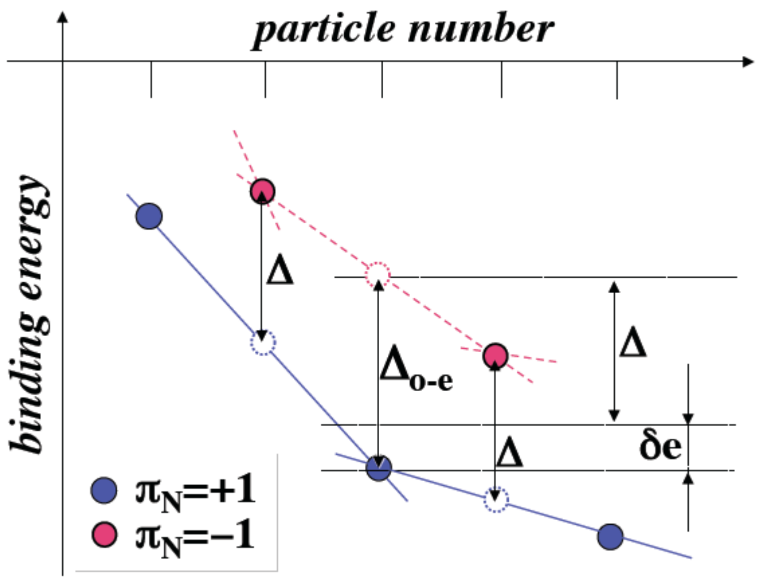
Two-neutron separation energy S_{2n}

$$\lambda = \frac{dB}{dN}$$

$$S_{2n}(N,Z) = B(N-2,Z) - B(N,Z) \approx -dB \approx -2\frac{dB}{dN} = -2\lambda_n$$

One-neutron separation energy S_{1n}

$$S_{1n}(N,Z) = B(N-1,Z) - B(N,Z)$$



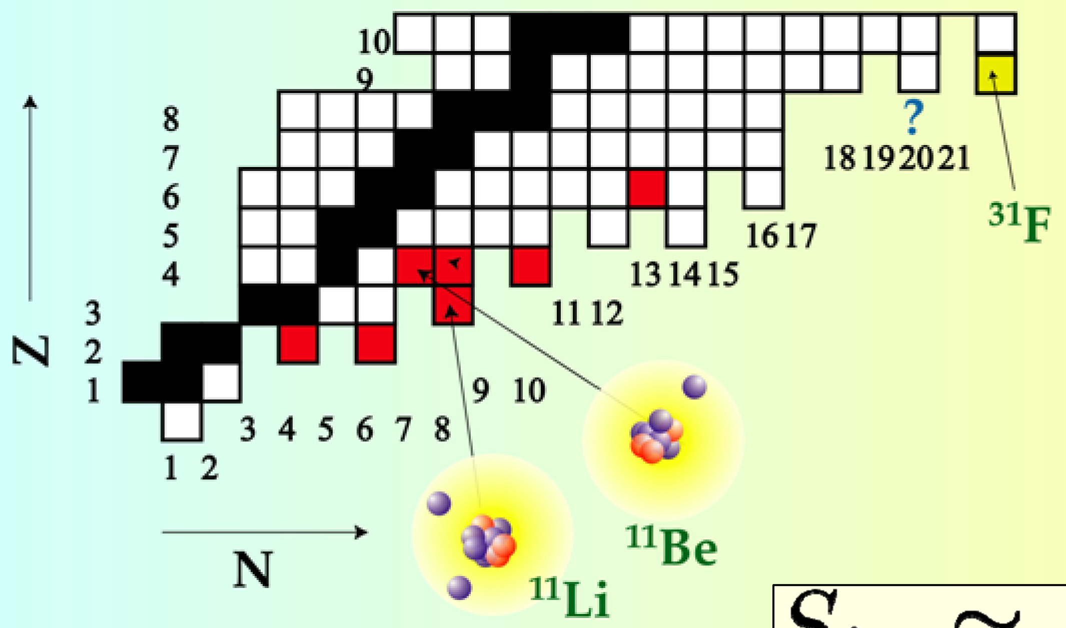
$$S_{1n} \approx -\lambda - \Delta$$ for odd-N

$$S_{1n} \approx -\lambda + \Delta$$ for even-N

$$S_{2n} = 0 \Rightarrow \lambda = 0$$

$$S_{1n} = 0 \Rightarrow \lambda = -\Delta$$

Light drip line nuclei



$$S_{1n} \approx -\lambda - \Delta$$

The single-particle field characterized by λ, determined by the p-h component of the effective interaction, and the pairing field Δ determined by the pairing part of the effective interaction are equally important when S_{1n} is small.

HFB theory in coordinate space

J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A422 (1984) 103

$$x = (\vec{r}, \sigma), \quad \int d^3\vec{r} \sum_{\sigma} \equiv \int dx$$

$$\int dx' \begin{pmatrix} h(x,x') & -\Delta(x,x') \\ -\Delta(x,x') & -h(x,x') \end{pmatrix} \begin{pmatrix} u(x') \\ v(x') \end{pmatrix} = \begin{pmatrix} E+\lambda & 0 \\ 0 & E-\lambda \end{pmatrix} \begin{pmatrix} u(x) \\ v(x) \end{pmatrix}$$

$$h(\vec{r}, \vec{r}') \to 0 \text{ and } \Delta(\vec{r}, \vec{r}') \to 0 \text{ for large } \vec{r}, \vec{r}'$$

$$\Rightarrow \begin{cases} -\frac{\hbar^2}{2M} \Delta u(x) = (\lambda + E) u(x) \\ -\frac{\hbar^2}{2M} \Delta v(x) = (\lambda - E) v(x) \end{cases}$$

$$u(x) \sim \begin{cases} r^{-1} \cos(k_1 r + \delta_1) & \text{for } \lambda + E > 0 \\ r^{-1} \exp(-\kappa_1 r) & \text{for } \lambda + E < 0 \end{cases}$$

$$v(x) \sim \begin{cases} r^{-1} \cos(k_2 r + \delta_2) & \text{for } \lambda - E > 0 \\ r^{-1} \exp(-\kappa_2 r) & \text{for } \lambda - E < 0 \end{cases}$$

$$\rho(x, x') = \sum_{0 < E_n < E_{\max}} v_n(x) v_n^*(x')$$

Localized!

- For λ>0 the entire spectrum is continuous.
- For |*E*|>-λ both components are localized

BCS gives wrong asymptotic behavior

If pairing is present, the picture of halo changes:
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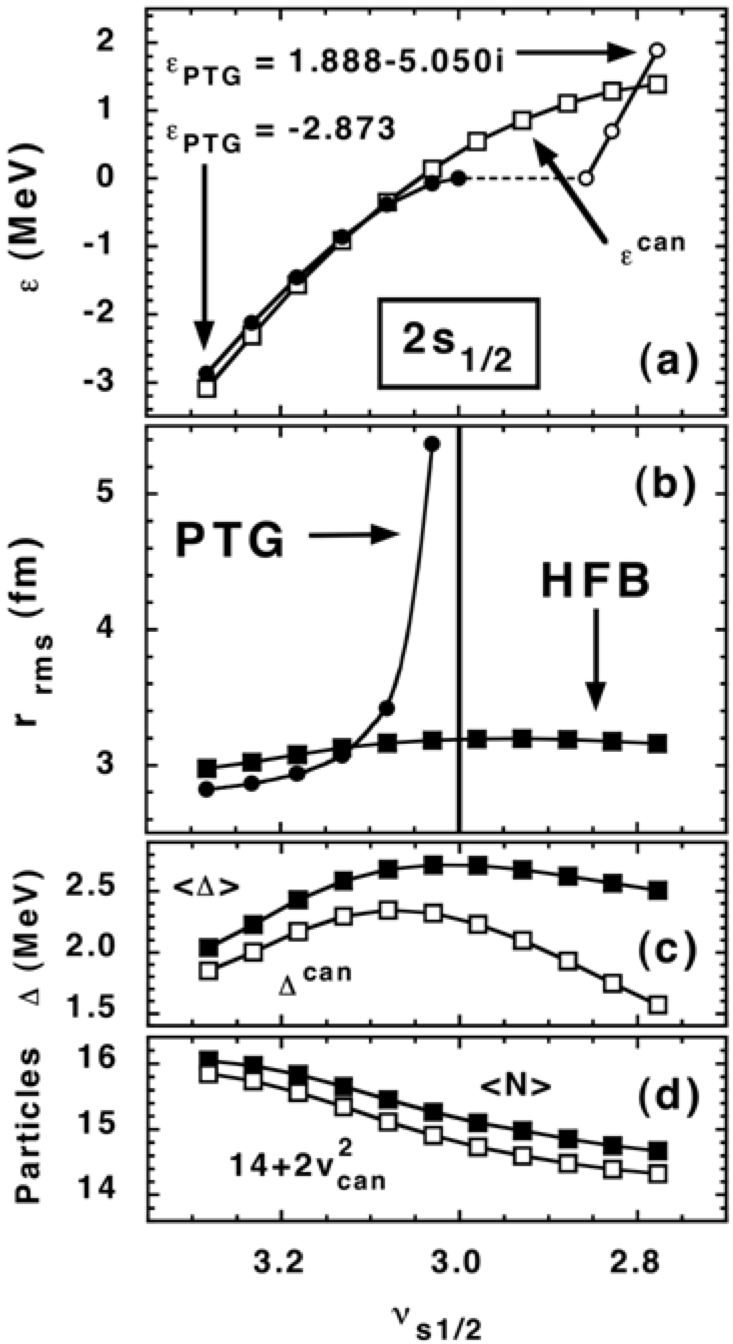


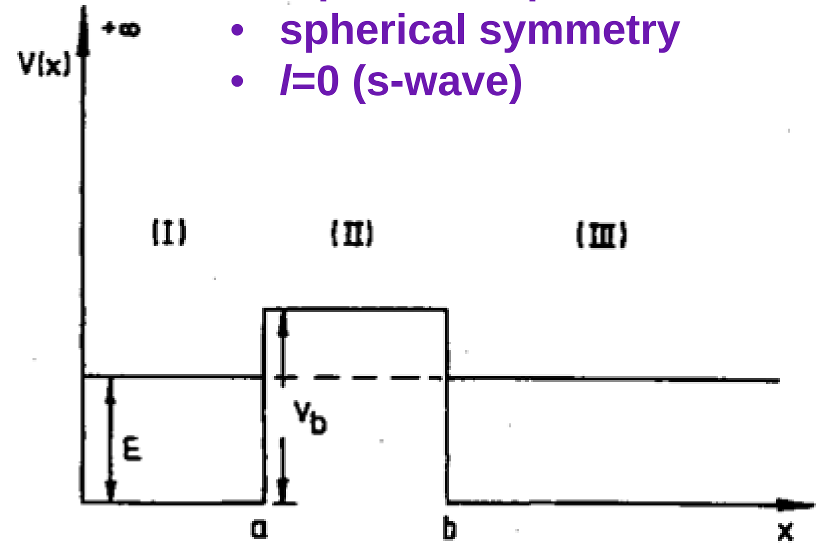
Pairing Antihalo Effect

When do resonances appear? A sqare well example

Radial Schrödinger equation

$$\chi'' + \frac{2M}{\hbar^2}(E - V)\chi = 0 \quad (\chi = \varphi r)$$

- square-well potential
- spherical symmetry
- *l*=0 (s-wave)



Region I:

$$\chi_I = A \sin pr, \quad p^2 = \frac{2ME}{\hbar^2}$$

Region II:

$$\chi_{II} = c_+ e^{q(r-a)} + c_- e^{-q(r-a)}, \quad q^2 = \frac{2M(V_b - E)}{\hbar^2}$$

Region III:

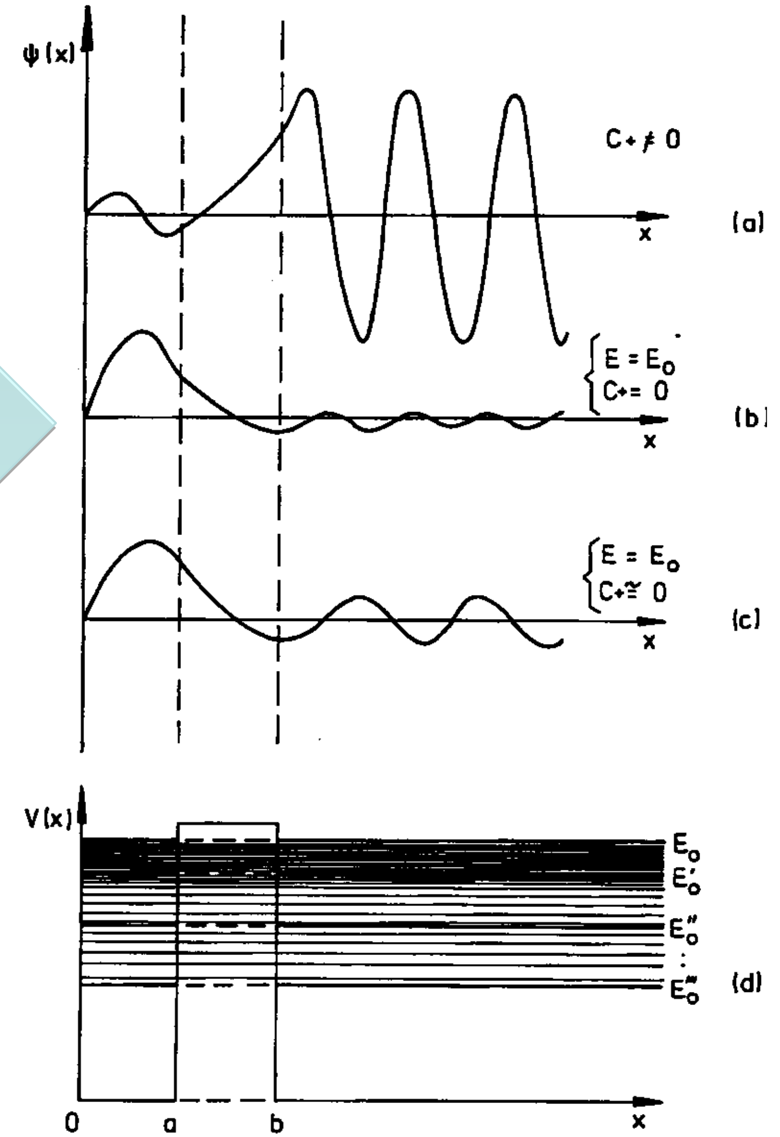
$$\chi_{III} = c_1 e^{ip(r-b)} + c_2 e^{-ip(r-b)}$$

In almost all cases $|\chi_{III}|$ is much larger than $|\chi_I|$. We are now interested in those situations where $|\chi_{III}|$ is as small as possible.

The condition

$$c_+ = 0 \Rightarrow \tan(pa) = -\frac{p}{q}$$

defines "virtual" levels in region I: particle is well localized; very small penetrability through the barrier



A comment on the time scale…

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$ TDSE

$$T_{1/2} = \ln 2 \frac{\hbar}{\Gamma}, \qquad \hbar = 6.58 \cdot 10^{-22}\ \mathrm{MeV} \cdot \sec$$

Can one calculate Γ with sufficient accuracy using TDSE?

$$T_{s.p.} \approx 3 \cdot 10^{-22} \sec = 3\,\mathrm{baby}\sec.$$

^{238}U: $T_{1/2}=10^{16}$ years

^{256}Fm: $T_{1/2}=3$ hours

For very narrow resonances, explicit time propagation impossible!